Chapter 4:- Quadratic Equations

QUADRATIC EQUATIONS

The polynomial of degree two is called quadratic polynomial and equation corresponding to a quadratic polynomial P(x) is called a quadratic equation in variable x.

Thus,

$P(x) = ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ is known as the standard form of quadratic equation.

There are two types of quadratic equation.

(i) Complete quadratic equation:

The equation $ax^2 + bx + c = 0$ where $a \neq 0$, $b \neq 0$, $c \neq 0$

(ii) Pure quadratic equation:

An equation in the form of $ax^2 = 0$, or $ax^2 + bx = 0$ where $a \neq 0$, $b = 0$, $c = 0$

ZERO OF A QUADRATIC POLYNOMIAL

The value of x for which the polynomial becomes zero is called zero of a polynomial.

For instance, 1 is zero of the polynomial $x^2 - 2x + 1$ because it became zero at $x = 1$.

SOLUTION OF A QUADRATIC EQUATION BY FACTORISATION

A real number x is called a root of the quadratic equation $ax^2 + bx + c = 0$, a 0 if $a\alpha^2 + b\alpha + c = 0$. In this case, we say $x = \alpha$ is a solution of the quadratic equation.

NOTE: -

1. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
2. Roots of quadratic equation $ax^2 + bx + c = 0$ can be found by factorizing it into two linear factors and equating each factor to zero.

SOLUTION OF A QUADRATIC EQUATION BY COMPLETING THE SQUARE

By adding and subtracting a suitable constant, we club the x^2 and x terms in the quadratic equation so that they become complete square, and solve for x.

In fact, we can convert any quadratic equation to the form $(x + a)^2 - b^2 = 0$ and then we can easily find its roots.

ROOTS OF THE QUADRATIC EQUATION

Let the quadratic equation be $ax^2 + bx + c = 0$ $(a \neq 0)$.
Thus, if $b^2 — 4ac \geq 0$, then the roots of the quadratic
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ equation are given by

QUADRATIC FORMULA

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ is known as the quadratic formula which is useful for finding the roots of a quadratic equation.

DISCRIMINANT

The expression $b^2 — 4ac$ is called the discriminant of the quadratic equation.

NATURE OF ROOTS

(i) If $b^2 — 4ac > 0$, then the roots are real and distinct.

(ii) If $b^2 — 4ac = 0$, the roots are real and equal or coincident.

(iii) If $b^2 — 4ac < 0$, the roots are not real (imaginary roots)

FORMATION OF QUADRATIC EQUATION WHEN TWO ROOTS ARE GIVEN

If α and β are two roots of equation then the required quadratic equation can be formed as $x^2 — (\alpha + \beta)x + \alpha\beta = 0$

NOTE :-

Let α and β be two roots of the quadratic equation $ax^2 + bx + c = 0$
then
Sum of Roots: – the coefficient of x / the coefficient t of x^2
$\Rightarrow \alpha + \beta = – b / a$

Product of Roots :-

$\alpha\beta$ = constant term / the coefficient t of x^2
$\Rightarrow \alpha\beta = c / a$

SOLUTION OF A QUADRATIC EQUATION BY DISCRIMINANT METHOD

Let quadratic equation is $ax^2 + bx + c = 0$

Step 1.

Find $D = b^2 — 4ac$.

Step 2.

(i) If $D > 0$, roots are given by $x = x = \frac{-b \pm \sqrt{D}}{2a}$

(ii) If $D = 0$ equation has equal roots and root is given by $x = -b / 2a$.

(iii) If $D < 0$, equation has no real roots.